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Pythagorean Theorem Concept Image in Junior High School: An Analysis in the Online-Based Learning

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Abstract*. The concept image comprises all the cognitive structures in an individual's mind connected with a specific notion. However, if the learning situation is not relevant to help students construct knowledge and provide meaningful experiences, it will affect their concept image. This study aims to examine the concept of the image of the Pythagorean theorem during online-based learning. The research is a qualitative approach, and the method is phenomenological to understand the meaning for the participants of their concept image. Initially, this study enrolled 66 students. Two students were selected as research subjects and acted as resource persons to provide adequate contextualization based on students who answered right and wrong. Data collection combines data from the outcomes of testing the Pythagorean theorem test, interviews, and literature studies. The data analysis technique employed is Interpretative Phenomenological Analysis (IPA). There are six categories of student concept image. Based on in-depth interviews with two students who answered correctly and incorrectly, students were given inconsistent meanings of the Pythagorean theorem due to the learning situation to improve students' understanding of the subject.*

Keywords*: concept image, pythagorean theorem, online-based learning, junior high school*

Introduction

Mathematics education aims not only to mastery of concepts but also to develop mathematical thinking skills (Suryadi, 2015b). Learning mathematics does not build meaning from mathematical concepts because it is considered a product ready to be taught (Maulida, 2018). Understanding mathematical ideas do not mean knowing information but can interpret interpreting and transform it into more meaningful so that it is helpful for students when solving more complex math problems (Jatisunda, 2019). Learning mathematics is meaningful if students reflect on issues, not in line with previous expectations and experiences (Cobb, 1994). Learning mathematics through a deep understanding and meaning process can be a provision for students' lives.

The current pandemic condition has resulted in online-based learning. Online learning is distance learning that utilizes computer technology, computer networks, and or the internet. Based on the research results, online-based learning positively impacts student learning success (Merchant et al., 2014; Schmid et al., 2009). Technology integration can improve academic performance, inspire and encourage better learning (Curtain, 2001). However, successful distance learning is very dependent on the teacher's understanding of managing the learning process (Leontyeva, 2018). Good conceptual knowledge and procedural knowledge from mathematics teachers positively impact the learning process in the classroom (Shulman, 1987). Conceptual knowledge and procedural knowledge are part of the knowledge of mathematics (Attorps, 2006). The online-based learning process will run well when the teacher can manage and teach the correct mathematical concepts according to students' needs.

Online-based learning takes into account not just the teacher but also the student. Prior experience enables students to comprehend better the things they are about to learn. The student thinks that when one hears the name of a concept, what typically comes to mind is not its definition but what he refers to as one's concept image (Dahl, 2017). Concept image is cumulative, changes over time, and is not simply a static item stored in memory, and particular aspects may be activated in response to some questions or problem situations, while other aspects are not (Wawro et al., 2011). Concept image is all mental pictures (pictorial, symbolic, and others), all mental attributes (conscious or unconscious), and associated processes are included in the concept image (Semadeni, 2008; Tall & Vinner, 1981a; Vinner, 1983). A concept definition is a formal description of a particular mathematical notion (Giraldo et al., 2003; Tall & Vinner, 1981a). Structural inequities and disparities in mathematical aptitude can be exacerbated when mathematics is not taught following students' experience requirements. The concept image and concept definition are the fundamental constructs of the cognitive growth process of students (Vinner, 1983). Thus, it is critical to pay attention to the design of online-based learning.

The concept image embodies the student's learning experience and serves as the foundation for students' ability to grasp new concepts. However, students' concept images often differ substantially from concept definitions (Lithner, 2011), and understanding a concept's definition in no way implies comprehension of the concept (Attorps, 2006; Vinner, 2002). It occurs because of students' learning processes, ways of thinking, conceptions, and difficulties, and the strong influence of the constructivist perspective they have been through (Artigue, 2016). Students' concept images will differ according to their experience learning process, without exception, when learning online-based learning.

In the design of teaching online-based learning, teachers should see students in terms of their strengths, not their weaknesses, and resist the temptation to quickly re-teach all skills that we believe students should have acquired during face-to-face meetings (NCTM, 2020). It is done to guarantee that the concept image and definition are identical. Because, at its core, the learning design must enable students to get a more formal grasp of concepts by building a concept image (Tall & Vinner, 1981a), and student experience greatly determines the readiness of students to learn at online-based learning (NCTM, 2020). However, when the concept image and definition are incoherent, it becomes difficult for students to see and work on developing a more profound knowledge (Burrill, 2019; Tall & Vinner, 1981a). As a result, student concept images do not correspond to scientific concepts. (Maulida, 2018).

Teaching design is fundamental to a teacher's professional activity by giving examples of various mathematical problems (Green & Olsher, 2018). It provides experience to form a more coherent understanding of the concept (Tall & Vinner, 1981a). However, when it comes to requiring online learning, teachers cannot comply, even if they are uninterested in online education (Cutri et al., 2020), and the shift to online-based learning can potentially alter teachers' teaching design (Cutri & Mena, 2020). Due to restricted internet connection and technology, most online-based learning activities in Indonesia utilize WhatsApp Groups. Several conclusions from implementing online-based learning include: teachers reported that while students opened the WhatsApp messages, they lacked the necessary data to download the resources, and teachers distribute instructional materials in photographs of book pages, workbooks, and short videos, among other formats. However, teachers discovered that many lacked the data necessary to download resources. The research findings indicate that the teacher has no way of knowing whether pupils are engaged in learning via WhatsApp Groups (Chirinda et al., 2021). As a result of the numerous restrictions associated with the implementation of online-based learning, the process of delivering subject matter is not well received by students, resulting in gaps in concept images and concept definitions.

The Pythagorean theorem remains a complex problem. There are still numerous errors committed by a student in junior high school when attempting to solve issues that have little effect on low learning outcomes (Yadrika et al., 2019). It is possible because students' experience and grasp of introductory concepts and the Pythagorean theorem are limited. Several research findings indicate that: Students' errors in completing Pythagorean Theorem-related questions and difficulties applying mathematics concepts (epistemological barrier) resulted from an insufficient comprehension of the concept, which rendered them unable to solve problems in various circumstances (Hutapea et al., 2015). Students with limited abilities comprehend the Pythagorean Theorem concept, which benefits students who struggle to solve problems effectively (Rachmawati, 2017). The study highlighted didactic and epistemological learning obstacles: an inability to comprehend mathematical techniques involving the Pythagorean theorem and algebraic ideas (Rahayu et al., 2021). Even after years of face-to-face instruction, the student still struggles to grasp the Pythagorean theorem notion. Additionally, when circumstances necessitate online learning and teachers rely solely on WhatsApp groups, the gaps between concept image and concept definition will widen. Based on this description, this research examines students' experiences when they carry out online-based learning with a specific assessment of the concept image of the Pythagorean theorem concept.

Method

Qualitative approaches, in general, are concerned with how humans perceive, describe, and understand particular phenomena (Creswell, 2012) by ascribing meaning to the phenomenon. Thus, the objective of these techniques is to generate a detailed account of some component of human experience (Qutoshi, 2018). The present study used Phenomenological research to understand the meaning for the participants of their learning experiences. Rather than testing a hypothesis, phenomenology studies rely on participants' expertise in the field of their own lived experience to provide specific insight into the phenomenon under inquiry (Smith, 1996). Due to the restrictions of the learning infrastructure, the learning process was conducted using WhatsApp Groups; zoom and google meet were unable to be used. Thus, in the context of this research's learning experience, it is online-based learning, specifically via WhatsApp groups. The teacher-designed learning environment begins with a greeting and introduction of the primary topic studied by pupils, followed by the arranging of activities, namely 1). Students should be instructed to read and comprehend the text appropriately, 2). Students should be instructed to study instances of questions. 3). Instructing students to create a summary of the subject covered, 4) and finally, instructing students to complete practice questions. Then, include links to resources linked to the concepts being studied in pdf files or YouTube videos.

The research took place in SMP Negeri 1 Sindang in Majalengka Regency. Participants are 66 students who have studied the Pythagorean theorem concept. Additionally, there is no force involved in subject selection; instead, subjects are chosen voluntarily. Then, two students were chosen as the research subjects and acted as interviewees because a small sample size can provide a sufficient viewpoint when contextualization is adequate (Chapman & Smith, 2002). In the phenomenological approach, random or deliberately selected samples are used (Lester, 1999). The researchers use a purposive sampling strategy to identify participants who share a common experience but differ in other ways. Participants in phenomenological research must be able to clearly describe, relate, and explain something or an event in order toto get an understanding and meaning of the event.

They are collecting qualitative data via observations, discussions, interviews, public records, and respondents' diaries, as well as researcher reflections (Creswell, 2012). The research technique used in this study is to combine data from the outcomes of testing the Pythagorean theorem test instrument material, observations, interviews, documents, and literature studies. Observations were conducted to observe teachers' delivery of online-based learning in detail. When the responder had completed working on the test instrument, interviews were performed. The interview approach employed is unstructured and informal, allowing for greater flexibility. Interpreting students' experiences following specialized online-based learning in the study of the Pythagorean theorem's concept image and concept definition are studied.

The phenomenological analysis is more appropriate to unravel the problems of human subjects, which are generally changing. Thus, this analytical approach uses an inductive logic system, where a syllogism is built based on specific things or field data that leads to general items. Qualitative analysis is generally not used to find data in terms of frequency but is used to analyze the meaning of data that appears on the surface-interpretative phenomenological analysis as written by (Smith et al., 1999). The stages of interpretative phenomenological analysis carried out are as follows: 1) reading and re-reading; 2) initial notes; 3) developing emergent themes; 4) searching for connections across emergent themes; 5) moving the following case, and 6) looking for patterns across cases.

Results and Discussion

The analysis of the data findings begins by explaining the concept image of the Pythagorean Theorem according to students' perceptions. Students with a range of mathematical abilities are invited to participate, of course, diversifies the idea image of the Pythagorean theorem. Then it is continued by describing the results of students' experiences, especially relating to the way students obtain the meaning of the concept of the pythagorean theorem. The distribution of student responses definition pythagorean theorem is presented in Table 1.

Table 1. Student concept image about pythagorean theorem

No	Concept Image	Number of Students
1	How to find the sides of a right triangle	9
2	Rules for determining the length of one side of a right triangle	18
3	How to calculate the side lengths of a right triangle where the other sides are known	10
4	How to calculate the sides of a triangle	15
5	The square of the hypotenuse of a right triangle is equal to the sum of the courts of the other two sides	8
6	No answer	6
	Total Students	66

The meaning of the concept of the pythagorean theorem is mainly written by students, namely in the second category of meaning, namely the rules for determining the length of one side of a right triangle. In general, students work on solving problems in accordance with procedures based on definitions. First they determine which side of the triangle they want to solve, then enter the values of the two sides that they already know into the Pythagorean formula, finally they complete the calculation to get the solution of the Pythagorean equation. However, in-depth interviews and additional analysis revealed that students provided inconsistent and imprecise responses regarding the definition of the Pythagorean theorem notion. Below is an example of student responses related to the answers to the questions written. Student responses related to this can be seen in Figure 1. and Figure 2.

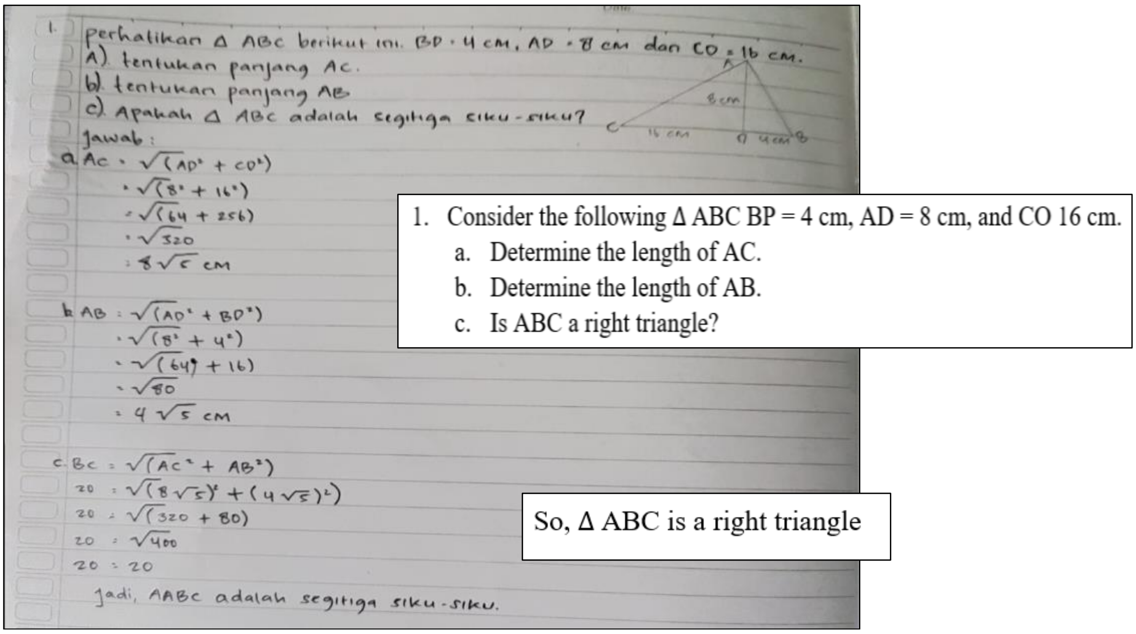

Figure 1. Students who answered correctly

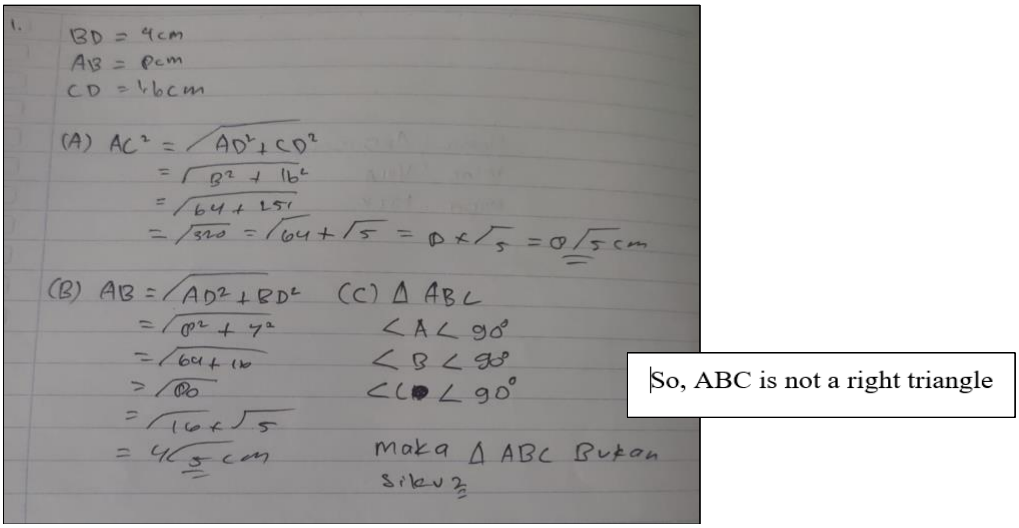

Figure 2. Students who answered wrong

The correct response demonstrates the student's procedural knowledge in addressing the provided problem, whereas the incorrect answer demonstrates procedural flaws in solving the problem. However, in applying the rules of the Pythagorean theorem, the procedural knowledge of the two students was good, as evidenced by using these rules. Then the researcher explores the definitions that the students have written by interviewing several students. The results of interviews it was found that the meaning of the Pythagorean theorem concept. Most students answered how to calculate the sides of a triangle, and to deepen students' understanding, interviews were conducted again. The results of the interviews are presented below.

Researcher	:	*What was the first thing that came to mind when your teacher assigned you to learn about the Pythagorean theorem via a WhatsApp group?*
Student	:	*Usually, the teacher gives assignments from the usual student books. However, we rarely read and learn directly from books. We immediately see assignments and work on questions, and we browse because every assignment is usually already on google.*
Researcher	:	*Try to state the meaning of the Pythagorean theorem according to your understanding!*
Student	:	*To calculate the sides of a triangle*
Researcher	:	*Try to explain how to calculate it! Give an example!*
Student	:	*"c" squared equals "a" squared plus "b" squared*
Researcher	:	*Suppose the length of the hypotenuse of a right triangle is 34 cm. The length of the side of the right side is 16 cm and x cm. determine the value of x!*
Student	:	$c^2 = a^2 + b^2$ --- $c^2 = 34^2 + 16^2 = 1156 + 256 = 1412, c = \sqrt{1412}$

Students' understanding based on interviews provides information that students do not fully understand the concept of the Pythagorean theorem. When given different questions, it is difficult to solve the problem. It occurs because the learning situation is not relevant to students' knowledge and understanding (Gagatsis & Patronis, 1990), and teachers play an essential role in creating appropriate situations in the learning process (Brousseau, 1976). Irrelevant situations will result in learning obstacles (Brousseau, 2006). However, it is not a weakness when students get obstacles. It indicates that students carry out the thinking process and become a reflection tool for teachers to improve learning because obstacles are student abilities to respond to observable and evaluated learning settings (Gagatsis & Kyriakides, 2000). Students' obstacles will eventually serve as an advantage for teachers in designing more relevant learning situations for students to create knowledge of the Pythagorean theorem concept.

The following describes the learning environment in which students were taught the Pythagorean theorem concepts 1) encouraging students to read and examine the item before opening it. 2) direct students to do the textbook activities. The researcher's investigation of the current situation can result in various perspectives of the Pythagorean theorem notion. According to student interviews, when instructed to complete practice questions from the textbook, most students did not respond based on what they had learned in the book. However, Google offers all of the information necessary to answer the arranged practice questions. As a result, the created circumstance does not fully assist pupils in understanding the Pythagorean theorem notion. The researchers hypothesize that students respond to questions based on the examples they read without attempting to comprehend them.

As a result of the learning situation and causing learning obstacles, other findings result in limited meaning related to concept image. It results in a gap between concept image and concept definition. The learning process in class as early as possible to introduce the concept image must align with the concept definition (Tsamir et al., 2015) because concept image and concept definition is basic constructions of students' cognitive structures (Vinner, 1983). Symbols, theorems, representations, properties, or other aspects related to concepts are part of the mental structure (Barnard & Tall, 1997). Good management between these parts of the cognitive system is essential for developing solid mathematical thinking (Giraldo et al., 2003).

Concept image is a mental structure of a mathematical concept that is built through student experience (Dreyfus, 2014; Tall & Vinner, 1981a; Viholainen, 2008), and a concept definition is a formal definition that describes a mathematical concept explicitly (Giraldo et al., 2003; Tall & Vinner, 1981). Concept image may not fully reflect the concept definition; concept image develops as students discover new mathematical ideas ((Engelke Infante et al., 2018; Habineza, 2013). The findings indicate that students do not possess sufficient knowledge of a concept's definition, implying a restricted concept image. It makes it harder for students to grasp and apply the concept definition (Clements & Sarama, 2011). Thus, teachers must create a learning situation that enables students to construct knowledge and maximize their potential. The relevant experience received by pupils due to the learning process is related to the conceptual image of a mathematical topic. Of course, a teacher's expertise in improving students' comprehension can help improve teaching approaches for correcting concept picture problems (Vinner, 2002). Thompson examines the impact of teachers' ideas on their instructional practices, and His research demonstrates that instructors' knowledge of mathematics affects what they do in the classroom. (Maulida, 2018). When a teacher cannot communicate a subject effectively, it affects students' comprehension. It causes students' concept images to diverge from scientific conceptions and conform to the study's conclusions.

The meaning of students on the concept of the Pythagorean theorem when viewed based on the didactic situation has not yet reached the institutionalization situation (Brousseau, 2006). Based on Plato's point of view, knowledge must pass through three categories: being true, believing in the truth, and proving the truth (Turri, 2012). Students are at the stage of knowing the truth and believing in it without verifying it. When the interview, students make procedural errors when solving the given problem. It can happen due to a lack of understanding and experience of students in using the Pythagorean rules. The means that students' knowledge construction is still at the information stage, not yet at the knowledge stage (Uriarte, 2008), where students have not reached the stage of justification of knowledge (Hofer & Pintrich, 1997). The knowledge possessed and obtained by students through learning has not provided sufficient experience to understand the concepts being taught, so that inside that truly belongs to itself has not been created. It has an impact on student learning outcomes (Schunk, 2012).

The concept image view based on knowledge theory is inseparable from two types of knowledge: tacit knowledge and explicit knowledge. Tacit knowledge is personal, and it is found in the individual's mind, which is an accumulation of learning and experience and goes through the process of interaction with others while explicit knowledge includes all kinds of information that comes from outside the individual and is systematic (Anumba et al., 2008; Herschel et al., 2001; Virtanen, 2013). It means that the concept of image is heavily influenced by tacit knowledge. Furthermore, students' conceptual and procedural understanding of mathematics is influenced by tacit knowledge (Frade & Borges, 2006). Based on the research findings, there is a possibility that students' conceptual and procedural knowledge cannot be used to solve problems because students' tacit knowledge is still low. Tacit knowledge and explicit knowledge have a role in students' knowledge construction, and good tacit knowledge helps students more easily understand explicit knowledge, and explicit knowledge becomes a means of developing tacit knowledge. However, online-based learning was not carried out.

Numerous research has been conducted on students' idea images, even though learning does not take place online or on the concept of the Pythagorean theorem. However, the study's findings indicate that there is a disconnect between concept image and concept definition. It occurs because the adopted learning method could not provide pupils with the experience and understanding necessary to create their knowledge in its entirety. According to the findings of the elementary school level research, the fourth elementary school kids' concept image of the platform is far from the ideal concept; this is because classroom learning is not age-appropriate (Saputro, 2020). Then, in junior high school, students' triangle concept images diverge from scientific concepts, and students do not entirely comprehend the significance of what they have studied (Haryati, 2020). Other research on the inequality of a single variable demonstrates that students' concept images are pretty distinct from teachers and mathematicians (Fatio, 2020). The students' conceptual image of the pyramid shape does not conform to the concept definition (T. Rahayu & Alghadari, 2019). When students comprehend integer operations, functions, and the idea of congruence, their conceptual images create learning difficulties (Dedy & Sumiaty, 2017). These investigations bolster the study's findings, particularly the discrepancy between concept image and concept definition.

Additionally, there is an inconsistency in the meaning of the Pythagorean theorem's concept, which is a strong indicator of a cognitive conflict known as compartmentalization. A phenomenon develops when pupils' cognitive structures contain two distinct and potentially contradictory conceptual schemas (Seel, 2011). A substitute is required to bridge the gap between concept image and concept definition, preventing compartmentalization. When assessing learning situation, the principles of Didactical Design Research (DDR), as demonstrated in the tripartite teacher-student-material interaction, serve as a frame of reference (Suryadi, 2019), and the subjective process of conceptualization (concept image) leads to a formal-universal institutionalization context (concept definition) or vice versa. (Suryadi, 2015a; Vinner, 1983). As a result, the design of learning situations plays a critical role in developing conceptual meaning during the mathematics learning process. Didactical Design Research (DDR) is a term that refers to a theoretical framework, conceptual framework, and methodological framework used in the development of instructional materials (Suryadi, 2010). According to the DDR approach, the character of reality in mathematics education manifests itself in various ways (Suryadi, 2019).

Concept images analysis of students can provide a snapshot of their understanding of a concept (Tall & Vinner, 1981b). Students' idea images are generated as a result of their educational experiences. The student learning experience occurs as a result of the learning situation's design. DDR can be used to close the gap between the concept image and the concept definition. Before becoming a learning situation design, DDR undergoes three steps of analysis: 1) prospective analysis, 2) metapedadidactic analysis, and 3) retrospective analysis. The prospective analysis step attempts to examine the teacher's design of the learning setting to ascertain the teacher's thought patterns when designing learning activities. At this step, the metapedadidactic analysis stage analyzes learning activities, conceptual errors, and students' thinking patterns. The purpose of retrospective analysis is to examine the achievement of learning objectives, find concept images, identify learning impediments, study hypothetical learning trajectories, and make recommendations for constructing learning circumstances.

The theoretical implication of this research is that the gap analysis between concept image and concept definition gives an overview of the meaning and comprehension of students' Pythagorean theorem concepts in the context of learning situation design. The practical implication of this research is that learning scenarios should be designed after extensive analysis or study to optimize the learning process and close the gap between concept picture and concept definition.

Conclusion

Students primarily write the definition of the Pythagorean theorem, especially the principles for determining the length of one side of a right triangle. However, according to student interviews, the Pythagorean theorem notion means how to compute the sides of a triangle. The meaning of the Pythagorean theorem thought that is produced is not scientific.

There are six categories of student concept image, namely: how to find the sides of a right triangle, rules for determining the length of one side of a right triangle, how to calculate the side lengths of a right triangle where the other sides are known, how to calculate the sides of a triangle, the square of the hypotenuse of a right triangle is equal to the sum of the courts of the other two sides, and no answer. Based on in-depth interviews with two students who answered correctly and incorrectly, students were given inconsistent meanings of the Pythagorean theorem due. It indicates that it occurs between the idea image and the definition of the concept. It occurs because the learning environment was not designed so that pupils could construct the Pythagorean theorem concept. The use of Whatsapp groups facilitates the online or offline learning process. If the learning environment is designed to think and create their knowledge, this will positively affect students' conceptual comprehension. This research is limited to analyzing the Pythagorean theorem's idea description. Additionally, the study did not explore the effect of concept drawings and definitions on the design of the following learning situations. Further research should focus on the most critical aspect of the Pythagorean theorem, namely the proof. The following advice is to investigate a hypothetical learning trajectory, analyze potential impediments to learning, and develop appropriate learning circumstances to replace the notion of drawing with the concept of definition.

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